

# Week 1: The existential theory and Kenny's challenges

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## 1 The development of modal logic

The development of modal logic as a discipline led to hopes that the technical advances in modal logic could serve as the basis for the theory of modal words in natural language.

Modal logics are usually defined on a language like this:

$$\mathcal{L} ::= p \in At \mid \varphi \wedge \psi \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid \diamond\varphi \mid \Box\varphi$$

$\Box$  and  $\diamond$  are treated essentially as quantifiers over classical models:  $\diamond\varphi$  says there's some classical model where  $\varphi$  is true, while  $\Box\varphi$  says  $\varphi$  is true at every classical model.

Things get slightly more complicated because possibility and necessity are *relative*: *that it's raining on earth* might be possible in a world like ours but not in a world where earth has the climate of Mars. So we make possibility and necessity *world relative* by (potentially) limiting which worlds a world can "access": we say  $w$  accesses  $w'$ , written  $wRw'$  or  $Rww'$ , iff  $w'$  is possible, relative to  $w$ , in the relevant sense.

Then we say  $\Box p$  is true at  $w$  iff  $p$  is true at all  $w'$  such that  $wRw'$ ; and  $\diamond p$  is true at  $w$  iff  $p$  is true at some  $w'$  such that  $wRw'$ .

This *semantics* gives rise to a corresponding *logic*. Just as we can ask, in classical logic, which sentences are true in every model (varying the interpretation function), likewise we can ask, in modal logic, which sentences are true at every world in every model (varying the interpretation function, set of worlds, and accessibility relation). The *minimal* modal logic K obtained by looking at all possible variations in these is the closure of the following axiom schemas:

- all propositional tautologies
- K:  $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$

under the rules:

- Detachment: if  $p \rightarrow q$  and  $p$  are derivable, so is  $q$
- Necessitation: if  $p$  is derivable so is  $\Box p$

Stronger logics can be obtained by making assumptions about the accessibility relation in question. Any modal logic extending K is known as a *normal* modal logic.

So, maybe, actual earth can access worlds where it rains and worlds where it doesn't; dry earth can only access worlds where it doesn't rain.

For the rest:

- $\llbracket A \rrbracket^w = 1$  iff  $\mathcal{I}(A, w) = 1$
- $\llbracket p \wedge q \rrbracket^w = 1$  iff  $\llbracket p \rrbracket^w = \llbracket q \rrbracket^w = 1$
- $\llbracket \neg p \rrbracket^w = 1$  iff  $\llbracket p \rrbracket^w = 0$
- $\llbracket p \vee q \rrbracket^w = 1$  iff  $\llbracket p \rrbracket^w = 1$  or  $\llbracket q \rrbracket^w = 1$
- $\llbracket p \rightarrow q \rrbracket^w = 1$  iff  $\llbracket p \rrbracket^w = 0$  or  $\llbracket q \rrbracket^w = 1$

## 2 A default view: Can is $\diamond$

What's the point of this apparatus? One can study it from many vantage points, but one application is as a theory of meaning of phrases like 'it's possible that' and 'it's necessary that'.

Early in this milieu, it was suggested that 'can' could be viewed as a  $\diamond$ . Which  $\diamond$ ? One whose interpretation is determined by the *context*:

See also Hilpinen 1969.

An important use of [the modal words] is connected with the notions of an ability and of a disposition and with the verb 'can'. For example: Jones can speak German (=it is possible for Jones to make himself understood in German); Jones cannot speak German (=it is impossible for Jones to make himself understood in German). (von Wright, 1951)

To say that something can happen means that its happening is compossible with certain facts. Which facts? That is determined, but sometimes not determined well enough, by context. An ape can't speak a human language—say, Finnish—but I can. Facts about the anatomy and operation of the ape's larynx and nervous system are not compossible with his speaking Finnish. The corresponding facts about my larynx and nervous system are compossible with my speaking Finnish. But don't take me along to Helsinki as your interpreter: I can't speak Finnish. My speaking Finnish is compossible with the facts considered so far, but not with further facts about my lack of training. What I can do, relative to one set of facts, I cannot do, relative to another, more inclusive, set. Whenever the context leaves it open which facts are to count as relevant, it is possible to equivocate about whether I can speak Finnish. It is likewise possible to equivocate about whether it is possible for me to speak Finnish, or whether I am able to, or whether I have the ability or capacity or power or potentiality to. Our many words for much the same thing are little help since they do not seem to correspond to different fixed delineations of the relevant facts. (Lewis, 1976)

This became one part of a developing orthodoxy, exemplified by Kratzer 1977, 1981, that *all* modal words in natural language have as their meanings  $\diamond$  or  $\square$ , with variation in interpretation due to context.

## 3 A basic worry

Let's suppose Susie is an inexpert darts player; she hits a bullseye on every thousand throws. She is not very much in control of the outcome of the throw; whether she hits the bullseye is essentially random. What should we say about:

(1) Susie can hit a bullseye.

On the existential analysis there are two options:

- we hold fixed local facts about Susie, which are certainly *compatible* with her hitting a bullseye, so (1) comes out true. Problem: many

would hesitate to assert (1) about Susie. Sloganistically: ability requires more than possibility.

- we hold fixed those plus something more so that (1) comes out false. Problem: ‘Susie cannot hit a bullseye’ is not assertable when we leave it open that she will!

Compare the following, from Hackl 1998:

- (2) John can swim.
- (3) John can answer this question
- (4) This elevator can lift 1500 lbs.

Heim, attr. to Bittner

These seem to just require more than compatibility.

## 4 Kenny’s critiques

These points are impressionistic, and at most point out a challenge that needs to be answered by a defender of the existential modal semantics. Kenny mounts a much more ambitious argument that

unlike deontic, epistemic and doxastic logics, the logic of ability cannot be captured in a modal system with a possible world semantics.

Specifically, he means with the classical possible world semantics given above.

### 4.1 Preliminaries

Kenny starts by distinguishing his target *ability* (like the ability to speak German) from *disposition* (like the power of water to dissolve salt). Following Aristotle’s observations, a rough distinction is this: if the conditions of a disposition are all in place, it will be exercised; not so of ability. If you put salt in water, then *mutatis mutandis*, it dissolves. If you put me in Germany, I could still fail to speak German.

How do we move from English sentences with the schematic form:

- (5)  $S \{ \overset{\text{can}}{\text{is able to}} \} \varphi$ .

to sentences of our language of interpretation? The problem is that  $\varphi$  in the above schema is a *verb phrase* and ‘can’ is, superficially, a two-place operator taking as arguments a verb phrase and noun phrase; while we’re looking for something to translate as a unary propositional operator  $\diamond$ .

One option: just treat ‘S can  $\varphi$ ’ as shorthand for ‘It can be (S  $\varphi$ )’, i.e.  $\diamond(\varphi(S))$ . However, this obviously won’t work, since the following are plainly inequivalent:

- (6) John can stand while Ernie balances on his shoulders.

(7) Ernie can balance on John's shoulders while John stands.

So it looks like 'S can' needs to be treated as its own modal, for each S. So we have  $\diamond_a, \diamond_b, \dots$  where the subscripts indicate different agents.

What about the verb phrase? The standard contemporary linguistics of a sentence like 'Susie can hit a bullseye' would be:

(8) Susie<sub>i</sub> [can [*pro*<sub>i</sub> [hit a bullseye]]]

Where *pro* is an unpronounced pronoun which is *obligatorily co-indexed with the overt subject*.

So on this view, 'can' actually already is a propositional operator (with an additional NP argument).

With all that said, Kenny was working before this tradition, and went a different route: he converted the target of analysis from 'I can' to 'I can  $\left\{ \begin{array}{l} \text{bring it about} \\ \text{make it the case} \end{array} \right\}$  that', so that he had as a target a superficially propositional operator. An important question to keep in mind is then to what degree his arguments target 'can' vs. 'can bring it about that'.

## 4.2 S4

Kenny considers the claim that 'can' has the logic S4, obtained with a reflexive, transitive accessibility relation, comprising K closed under the additional axiom schemata

- T':  $p \rightarrow \diamond p$
- 4':  $\diamond \diamond p \rightarrow \diamond p$

Aka *Success*

Kenny argues against each of these in turn. Starting with 4':

An interpretation of this on the lines suggested would be 'If I can bring it about that I can bring it about that I am speaking German, then I can bring it about that I am speaking German' or, equivalently, 'If I can acquire the ability to speak German, I can speak German'. This is clearly false. If, when applying for a post in a German department, I am asked whether I can speak German, it would hardly be proper for me to reply 'yes', starting from the premise that I can acquire the ability to speak German (say by attending courses for three years) and reasoning with the aid of [4'] and modus ponens.

Turning next to T':

If I am speaking German, surely I can speak German. . . But is it so clear? Perhaps, we may imagine, it is inconceivable that someone should speak a language without being able to speak it. In fact, it is quite often done. The late Pope Pius XII used to give audiences to American servicemen at the Vatican. The gracious speech which he delivered on these occasions had been composed, I was told, by an Irish monsignore and

a topic we'll return to at greater length; this principle is often called *Success* in the ability literature.

learned by heart under the coaching of an elocutionist. At those audiences the Pope spoke English; but he was not, in the normal sense, able to speak English. . . A hopeless darts player may, once in a lifetime, hit the bull[s-eye], but be unable to repeat the performance because he does not have the ability to hit the bull. I cannot spell 'seize': I am never sure whether it is an exception to the rule about 'i' before 'e'; I just guess, and fifty times out of a hundred I get it right. On each such occasion we have a counter-example to [T]: it is the case that I am spelling 'seize' correctly but it is not the case that I can spell 'seize' correctly.

Counterexamples similar to these will always be imaginable whenever it is possible to do something by luck rather than by skill. *But the distinction between luck and skill is not a marginal matter in this context: it is precisely what we are interested in when our concern is ability, as opposed to logical possibility or opportunity. . .* Of course it is on the basis of people's performances that we attribute skills and abilities to them; but a single performance, however successful, is not normally enough to establish the existence of ability. (I say 'not normally' because a single performance may suffice if the task is sufficiently difficult or complicated to rule out lucky success. Pushing one's wife in a wheelbarrow along a tightrope stretched across Niagara Falls would be a case in point.) But it would only be if a single performance always established an ability that we could offer T' as a law of the logic of ability.

Although there are modal logics that don't include T', standard ways of thinking about the semantics of ability modals will include T', since standard specifications say that accessibility holds fixed certain *facts*. The only way T' can fail is if non-facts can be held fixed.

### 4.3 Against K

If we stopped here, we'd have interesting but not shocking arguments: that the logic of 'can' does not include T or 4. But there are plenty of normal modal logics that don't include these.

A much more striking argument comes when Kenny turns to principles which follow from K: for if those are invalid for 'can', it shows that 'can' is really something very different from the usual  $\diamond$ .

Kenny argues against two principles of K: first,  $\diamond p \rightarrow \diamond(p \vee q)$ :

The President of the United States has the power to destroy Moscow, i.e., to bring it about that Moscow is destroyed; but he does not have the power to bring it about that either Moscow is destroyed or Moscow is not destroyed. The power to bring it about that either p or not p is one which philosophers, with the exception of Descartes, have denied even to God.

This seems like a good argument against this principle *for 'can bring it about that'*, but it's not clear to me it extends to 'can'.

In the other direction, more strikingly, is an argument against  $\diamond(p \vee q) \rightarrow (\diamond p \vee \diamond q)$ :

Given a pack of cards, I have the ability to pick out on request a card which is either black or red; but I don't have the ability to pick out a red card on request nor the ability to pick out a black card on request. That is to say, the following ( $\diamond(r \vee b)$ ) is true:

- (9) I can bring it about that either I am picking a red or I am picking a black.

but the following ( $\diamond r \vee \diamond b$ ) is false:

- (10) Either I can bring it about that I am picking a red or I can bring it about that I am picking a black.

Similar counterexamples can be constructed in connection with any other discriminatory skill (e.g., one may have sufficient skill at darts to be quite sure of hitting the board, and yet not be at all sure of obeying either the command 'Hit the top half of the dartboard' or the command 'Hit the bottom half of the dartboard').

Does this work for 'can' as well? If so, then it cannot be given a semantic in terms of compatibility with accessible worlds, no matter how we think about that accessibility relation.

Kenny diagnoses the problem this way:

Now what would be the ... intuitive account of the [accessibility] relation for a logic of ability? One suggestion which comes to mind is that in the logic of ability  $w_2$  is alternative to  $w_1$  if in  $w_2$  all the abilities present in  $w_1$  have been exercised. Analogy with the other cases would suggest that if this were the appropriate relation then the 'can' of ability should be represented as a strong modal operator  $[\Box]$ , not, as we have so far supposed, a weak one like the other 'can's. At first sight this seems reasonable enough. But reflection shows that there is something wrong with the idea of a world in which all A's abilities are exercised. For suppose that for some  $\varphi$  A is able to  $\varphi$  and is able not to  $\varphi$ : John, say, can be a smoker and can also be a non-smoker, i.e., not be a smoker. Then in a world in which all John's abilities are exercised, it will be true both that John is a smoker and that he is not a smoker. And that is not a possible but an impossible world.

It is not surprising, it may be suggested, that the 'can' of ability should prove recalcitrant when considered as a modality: for it represents a complex concept where the theories of modality and of activity intersect. The way out of the difficulties, therefore, may be to separate out, in formalisation, the motions of possibility and action. Suppose, for instance, that instead of representing 'I can bring it about that...' by the operator  $[\diamond]$  alone, one introduced an operator 'D' corresponding to 'bring it about that...' so that 'I can bring it about that p' was symbolised by  $\diamond Dp$ .