Indicatives and context-sensitivity

Day 2 // Conditionals and Information-Sensitivity, ESSLLI // David Boylan and Matt Mandelkern

1 Or-to-if

From $p \vee q$ we can generally infer $\neg p >_i q$ when $\neg p$ is epistemically possible:

$>_i$ is the indicative conditional.

(1) Either the butler or the gardener did it. Both remain suspects.

$\rightsquigarrow$ So, if it wasn't the butler, it was the gardener.
$\rightsquigarrow$ So, if it wasn't the gardener, it was the butler.

This is something special about indicatives:

(2) $\not\rightsquigarrow$ So, if it hadn't been the butler, it would have been the gardener.

More examples:

(3) The die landed prime.

$\rightsquigarrow$ So, if it didn't land two or five, it landed three.
$\not\rightsquigarrow$ So, if it hadn't landed two or five, it would have landed three.

(4) It'll be Adams or Wiley.

$\rightsquigarrow$ So, if it's not Adams, it will be Wiley.
$\not\rightsquigarrow$ So, if it weren't to be Adams, it would be Wiley.

The inference doesn't seem to go through when the antecedent is ruled out:

(5) The butler did it. So, the butler or the gardener did it.

$\rightsquigarrow$? So, if the butler didn't do it, the gardener did it.

Indeed, the indicative here doesn't really seem assertable at all.

We might want to restrict the inference to non-modal p, q. Plausibly we should generally accept $\Box r \vee \neg r$:

(6) Either it must have been the butler, or it wasn't the butler.

And plausibly we leave open that it's not the case that it must have been the butler. But then we could infer:

(7) If it might not have been the butler, it wasn't the butler.

Likewise, suppose we don't know the color of a randomly chosen card. Then we accept that either it's red or it might be red, and leave open that it's not red, but can't conclude:

(8) If it is not red, it might be red.

2 Not an entailment

Why not just say: $p \vee q \models \neg p >_i q$?

- Suppose we also have MP, at least for non-modal p, q. Then we'd have $p \vee q \Dashv\models \neg p >_i q$: that is, the untenable material analysis.
- Either way, we'll have $\neg(p >_i q) \models p \wedge \neg q$, which is untenable, as we've seen.
- This point is somewhat ameliorated if instead we have $\Diamond p \wedge (\neg p \vee q) \models p >_i q$. Contraposing, we'd have $\neg(p >_i q) \models \Box\neg p \vee (p \wedge \neg q)$. Does 'It's not the case that, if Susie is a lizard, she is warm-blooded' entail 'Either Susie must not be a lizard, or else she is a cold-blooded lizard'? Maybe.

A more decisive arguments comes from probabilities, which don't track or-to-if in the way they generally track entailments. If $p \models q$, then $P(q) \geq P(p)$ for any probability measure P.

> It was either the gardener, butler, or chauffeur. There is a 70% chance it was the gardener, a 25% chance it was the butler, and 5% chance it was the chauffeur. Hence, I am confident in (9):
>
> (9) It was the gardener or chauffeur.
>
> But I doubt:
>
> (10) It was the chauffeur if not the gardener.

Of course, we could find notions of entailment which don't preserve probabilities or contrapose, and say or-to-if is valid. This is a somewhat unhelpful way of talking, though.

Instead, what seems true is that or-to-if is a *reasonable inference*, in the sense that if you *accept* $p \vee q$ and leave open $\neg p$, you can infer $\neg p >_i q$.

More contemporary terminology: it is *acceptance* or *informationally* valid.

Put another way: whenever the *context set* entails $p \vee q$ and is compatible with $\neg p$, it should also entail $\neg p >_i q$.

The context set is the set of worlds compatible with the common assumptions in the conversation. Stalnaker fleshes this out in terms of common acceptance, though other ways of spelling it out are compatible with the basic point.

Why think the context set is the relevant notion, rather than individual beliefs? Suppose I'm sure it was the butler. You're sure it was the gardener or the butler but leave it open it was the gardener. It seems like we should agree, between us, that if it wasn't the gardener, it was the butler, and I can use conditionals like:

(11) If it wasn't the butler, the gardener was in the house at the time. But the gardener wasn't in the house at the time. So it must have been the butler!

A different option (Dorr and Hawthorne, 2018): What is valid is

- Modal or-to-if: $\Box(p \vee q) \models \neg p >_i q$

Potentially close if $\Box$ goes with common ground.

3 Antecedent compatibility

Another distinguishing feature of indicatives is that they appear to require that their antecedents be epistemically possible:

(12) John lost the race.

a. #If he won, he was really happy.
b. If he had won, he would have been really happy.

(13) Latif is in the kitchen.

a. #If he is at the pub, I'll ask him to come home.
b. If he had been at the pub, I would have asked him to come home.

There are some intriguing counterexamples to this:

(14) Susie will be at the party.

a. If she isn't there, we'll see her after.
b. If she weren't to be there, we could see her after.

Though note:

(15) Susie will be at the party.

a. #If she won't be there, we'll see her after.
b. If she weren't to be there, we could see her after.

4 Stalnaker's proposal

Suppose we want to augment a Stalnaker/Lewis semantics so that (i) or-to-if is a reasonable inference for indicatives in the sense above and (ii) indicative compatibility holds.

- to acceptance-validate or-to-if, require: $\forall\varphi : \forall w \in cs : \varphi \cap cs \neq \varnothing \rightarrow f_i(\varphi, w) \subseteq cs$.

  Exercise: show that this corresponds to or-to-if.

f_i is the indicative selection function, and cs is the context set

- to validate indicative compatibility, require that $\varnothing \subsetneq f_i([\![p]\!], w) \subseteq cs$
- We need a sentence-relative notion here, since no frame condition can ensure that $\varnothing \subsetneq f_i([\![p]\!], w) \subseteq cs$ for all p. And if we have instead $\forall\varphi : \varphi \cap cs \neq \varnothing \rightarrow \varnothing \subsetneq f_i(\varphi, w) \subseteq cs$, this won't rule out using $p >_i q$ when $[\![p]\!] \cap cs = \varnothing$.

One thing Stalnaker says suggests this.

  - this suggests that what we need here is not a *frame* or *model* condition but rather a *sentence-relative* requirement: a condition for the use of a particular sentence
  - we might call it a *presupposition*
  - though it is different from standard 'semantic presuppositions'

⇝ it is not a content, rather a structural constraint

⇝ (hence) it doesn't have characteristic projection behavior of SPs

(16) If the closest Mark-win-worlds are contextually possible, then if Mark won, he is really happy.

(17) If Susie used to smoke, then she stopped smoking.

- That suggests we could reformulate the first: rather than a frame condition, we can simply say:
  - $p >_i q$ is only assertable in cs if, $\forall w \in cs : \varnothing \subsetneq f_i(p, w) \subseteq cs$.

this is weaker, but will make or-to-if a reasonable inference *in the cases we observe*.

A good argument for a notion of context set which is accessible to the grammar.

5 Context-sensitivity: indicatives vs. subjunctives

There's obviously context-sensitivity in the interpretation of conditionals.

The present considerations suggest that at least some of that context-sensitivity is grammaticalized, in the sense that the conditional's mood tells you what kind of accessibility relations are admissible, relative to the context's information.

6 Not enough?

This is Stalnaker's story, taken on and developed in different ways by many theories. But both indicative compatibility and or-to-if seem to extend to embedded environments in ways not predicted by Stalnaker's story.

- Conjunction:

(18) a. #Suppose that John came to the party and that if he didn't come, he went to work.
b. Suppose that John came to the party and that if he hadn't come, he would have gone to work.

(19) a. #Everyone who went to Germany and went to Paris if not Germany had fun.
b. Everyone who went to Germany and would have gone to Paris if they hadn't gone to Germany had fun.

- In general: $p \wedge (\neg p >_i q)$ and $(\neg p >_i q) \wedge p$ are infelicitous, whether embedded or not.

- Conditionals:

(20) We have a die weighted towards either odds or evens.
a. #If the die was thrown and landed four, then if it didn't land four, it landed two or six.
b. If the die had been thrown and had landed four, then if it hadn't landed four, it would have landed two or six.

- In general: $p >_i (\neg p >_i r)$ is infelicitous.

◦ Quantifiers:

(21) a. #Everyone who is wearing brown is wearing grey if not brown.
b. Everyone who is wearing brown would have worn grey if they hadn't worn brown.

- In general: $Q(p)(\neg p >_i q)$ is infelicitous.

◦ Attitudes:

Thanks to Kyle Blumberg, p.c.

(22) It didn't rain yesterday. But Susie doesn't know that. Susie thinks that, if it rained yesterday, the picnic was cancelled.

- In general: $A_S\,(p >_i q)$ requires only that p be compatible with A_S.

◦ Disjunction:

(23) a. #Either Susie won and if she didn't she was upset, or else Mark won and if he didn't he was upset.
b. Either Susie won and if she hadn't she would have been upset, or else Mark won and if he hadn't he would have been upset.

Likewise, or-to-if seems valid in embedded contexts:

◦ Attitudes:

Boylan and Schultheis 2021.

(24) Liz believes it was the gardener or the butler.
a. $\rightsquigarrow$ So Liz believes that, if it wasn't the gardener, it was the butler.
b. $\not\rightsquigarrow$ So Liz believes that, if it hadn't been the gardener, it would have been the butler.

- In general: From ⌜S believes $p \vee q$⌝ we can infer ⌜S believes $\neg p >_i q$⌝.

◦ Conditionals:

Gillies 2004.

(25) a. $\rightsquigarrow$If Sue or Mark was here, then if it wasn't Sue, it was Mark.
b. $\not\rightsquigarrow$If Sue or Mark had been here, then if it hadn't been Sue, it would have been Mark.

- In general: $(p \vee q) >_i (\neg p >_i q$ is valid.

◦ Quantifiers:

(26) a. $\rightsquigarrow$ Everyone who went to Germany or France went to Germany if not France.
b. $\not\rightsquigarrow$ Everyone who went to Germany or France would have gone to Germany if they hadn't gone to France.

- In general: $Q(p \vee q)(\neg p >_i q)$ is valid.

7 More information sensitivity

We need more information sensitivity than in Stalnaker's system.

7.1 Domain semantics

On Yalcin (2007)'s view, modals and conditionals are sensitive to a shiftable *information state* parameter.

An info state is a set of possible worlds. The Boolean fragment is standard:

- $[\![A]\!]^{s,w} = 1$ iff $w \in \mathcal{I}(w)$
- $[\![p \wedge q]\!]^{s,w} = 1$ iff $[\![p]\!]^{s,w} = [\![q]\!]^{s,w} = 1$
- $[\![p \vee q]\!]^{s,w} = 1$ iff $[\![p]\!]^{s,w} = 1$ or $[\![q]\!]^{s,w} = 1$
- $[\![\neg p]\!]^{s,w} = 1$ iff $[\![p]\!]^{s,w} = 0$

Modals and conditionals are sensitive to s. In particular,

- $[\![p >_i q]\!]^{s,w} = 1$ iff $\forall w' \in s_p : [\![q]\!]^{s_p,w'} = 1$ where s_p is the maximal substate of s which accepts p

That is, which is such that $\forall w' \in s_p : [\![p]\!]^{s_p,w'} = 1$. Problems about uniqueness here.

Attitude verbs shift this parameter:

- $[\![S_a p]\!]^{s,w} = 1$ iff $\forall w' \in S_{a,w} : [\![p]\!]^{S_{a,w},w'} = 1$
- $[\![B_a p]\!]^{s,w} = 1$ iff $\forall w' \in B_{a,w} : [\![p]\!]^{B_{a,w},w'} = 1$

This gets or-to-if under attitudes nicely. If $B_a(p \vee q)$ is true at $\langle s, w\rangle$ then $B_a(\neg p >_i q)$ is too.

And if we add a compatibility constraint, it gets some compatibility data. Say that $p >_i q$ is defined at $\langle s, w\rangle$ only if $s_p \neq \emptyset$. Now suppose $S_a(p \wedge (\neg p >_i q))$ is true. Then $\forall w' \in S_{a,w} : [\![p \wedge (\neg p >_i q)]\!]^{S_{a,w},w'} = 1$. But then $s_{\neg p} = \emptyset$, contrary to the definedness constraint.

This falls short, however, in quantified contexts and disjunctive contexts. The issue is that the info sensitivity here is only induced by attitude predicates.

So $(p \wedge (\neg p >_i q)) \vee (r \wedge (\neg r >_i s))$ is consistent, even with the compatibility constraint above. It is true, for instance, in an info state which has only $p\bar{r}s$-world and $r\bar{p}q$-worlds.

Likewise, $every_x(px, \neg px >_i qx)$ is consistent, on the natural extension of this system to quantifiers:

- $[\![every_x(p, q)]\!]^{g,s,w} = 1$ iff $\forall a \in D : [\![p]\!]^{g_{x \to a},s,w} = 1 \to [\![q]\!]^{g_{x \to a},s,w} = 1$

Just consider an s such that, for every $w \in s$ and $a \in D$, if $\neg p(a)$ is true at w, then $q(a)$ is, too.

8 Dynamic semantics

It looks like we need *more* information sensitivity, not just from intensional operators but also from connectives. This is something we get from dynamic semantics.

See Heim 1982 for the general framework and Dekker 1993; Groenendijk et al. 1996; Gillies 2004 for integrating modals and conditionals.

Sentence meanings here are functions from contexts (sets of worlds=info states) to contexts:

- $c[A] = \{w \in c : w \in \mathcal{I}(A)\}\}$
- $c[p \wedge q] = c[p][q]$
- $c[\neg p] = c \setminus c[p]$
- $c[p \vee q] = c[p] \cup c[\neg p][q]$
- $c[B_a p] = \{w \in c : B_{a,w}[p] = B_{a,w}\}$

Conditionals check the input contexts to see if they accept the consequent once updated with the antecedent:

- $c[p >_i q] = \begin{cases} c & c[p] = c[p][q] \\ \varnothing & otherwise \end{cases}$

Add a compatibility presupposition as for Yalcin:

- $c[p >_i q]$ is defined only if $c[p] \neq \varnothing$

For attitudes, things work much as for Yalcin. If $c[B_a(p \vee q)] = c$, then $c[B_a(\neg p >_i q)] = c$ where defined.

For connectives, things are better. For instance, $(p \wedge (\neg p >_i q)) \vee (r \wedge (\neg r >_i s))$ will be undefined for Boolean p, r. Likewise for quantifiers. On a very simple approach, treat contexts as pairs of an assignment and info state. Then:

- $c_g[everyx(p,q)] = \{w \in c : \forall a \in D : w \in c_{g_{x \to a}}[p] \to w \in c_{g_{x \to a}}[p][q]\}_g$

The problem with this approach is it predicts order-sensitivity which does not seem to be there:

(27) a. A student of mine won but is sad if they didn't win.
b. A student of mine is sad if they didn't win but they won.

Dynamic semantics predicts that the first of these is inconsistent while the second is consistent.

This shows up in various ways in embedding environments. In theory you can define operators in a way which bleaches out this order-sensitivity, but that is somewhat roundabout.

8.1 Symmetric domain semantics

We need something like the info sensitivity that dynamic semantics incorporates into connectives, but we need a symmetrical version of it.

A natural idea is to return to the Yalcin semantics, plus compatibility constraint, but let connectives shift the info state too:

- $[\![p \wedge q]\!]^{s,w} = 1$ iff $[\![p]\!]^{s_q,w} = [\![q]\!]^{s_p,w} = 1$
- $[\![p \vee q]\!]^{s,w} = 1$ iff $[\![p]\!]^{s_{\neg q},w} = 1$ or $[\![q]\!]^{s_{\neg p},w} = 1$

This does capture our data, but it has a few problems.

- s_p is not always uniquely defined. We could deal with this super- or subvaluationally, but something has to be said.
- Some curious properties. We have:
  - $[\![\Box p]\!]^{s,w}$=1 iff $\forall w' \in s : [\![p]\!]^{s,w'} = 1$

Then consider a state s with p and $\overline{p}$ worlds. Then $\Box p$ is false at $\langle s, w\rangle$ but $\Box p \wedge \Box p$ is true at $\langle s, w\rangle$ for any w. This is prima facie quite weird.

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