# Logic and information sensitivity

Day 3 // Conditionals and Information-Sensitivity, ESSLLI // David Boylan and Matt Mandelkern

## 1 Bounded conditionals

An alternative: use local contexts, not to give you the domain of quantification for modals/conditionals, but rather to *bound* the admissible domains.

That is, adopt something like the architecture above, but view the info states in the way that, roughly, Karttunen (1973) did, as inputs for a secondary algorithm which computes whether a sentence is assertable ("satt").

That is, where $\kappa^p$ is the set of worlds in $\kappa$ where $p$ is true and satt relative to $\kappa$, we say:

- a sentence $p$ is satt in context $c$ iff it's satt relative to $\kappa_c$, the context set of $c$
- if $A$ is atomic, $A$ is satt at $\kappa$
- $p \wedge q$ is satt at $\kappa$ iff $p$ is satt at $\kappa^q$ and $q$ is satt at $\kappa^p$
- $p \vee q$ is satt at $\kappa$ iff $p$ is satt at $\kappa^{\neg q}$ and $q$ is satt at $\kappa^{\neg p}$
- $\neg p$ is satt at $\kappa$ iff $p$ is satt at $\kappa$
- $A_S(p)$ is satt at $\kappa$ iff $\forall w \in \kappa : p$ is satt at $A_{S,w}$

With this architecture in hand, say that conditionals have the *truth-conditions* of Stalnaker's conditional, but they also have non-trivial satt conditions:

- *Conditional Locality*: $p >_i q$ is satt at $\kappa$ only if $\forall w \in \kappa : \varnothing \subsetneq f_i(\llbracket p \rrbracket, w) \subseteq \kappa$

plus $\kappa \neq \varnothing$, plus $p$ is satt at $\kappa$ and $q$ at $\kappa^p$.

This looks just like Stalnaker's constraint. The difference is that, instead of interpreting $\kappa$ as the context set, as in Stalnaker's system, it's specified recursively as above.

The result is that a sentence containing $p >_i q$ won't be satt when the local context of $p >_i q$ entails $\neg p$. For instance, $\neg p \wedge (p >_i q)$ cannot be satt (whether embedded or not). Importantly, likewise for $(p >_i q) \wedge \neg p$.

And, if a local context entails $p \vee q$, then $p >_i q$ will be true throughout that local context (if it's satt), capturing local or-to-if. For instance, if $B_a(p \vee q)$ is true at $w$, then $B_a(\neg p >_i q)$ is true at $w$ if it is satt at any set of worlds containing $w$.

Bounds don't influence truth-values; so this change does nothing to the underlying logic; for instance, $p$ is always logically equivalent to $p \wedge p$.

## 2 Qualitative Collapse Results

Jointly adopting certain logical principles trivializes our theory of the conditional in various ways.

The connection to info sensitivity: these collapse results often turn centrally on the *Import-Export* principle, which looks like an encoding of some kind of info sensitivity:

- Import-Export (IE): $p > (q > r) \dashv\models (p \wedge q) > r$

I use > for both the indicative and subjunctive conditional.

Usually, the trivial conclusion is that $a > c \dashv\models a \supset c$: the conditional "collapses" to the material. This conclusion 'trivializes' our models in that it shows them to be equivalent to the material conditional.

Dale 1974, 1979

Suppose in particular we adopt the following in addition to IE:

- Modus Ponens (MP): $\{p, p > q\} \models q$
- Logical Implication (LI): if $\models p \supset q$ then $\models p > q$

Then we get collapse:

1. $p > q \models p \supset q$ — MP, conditional proof
2. $\models (\neg p \wedge p) > q$ — classical logic, LI
3. $\models \neg p > (p > q)$ — IE, 2
4. $\models \neg p \supset (p > q)$ — 3, MP
5. $\models (q \wedge p) > q$ — classical logic, LI
6. $\models q > (p > q)$ — IE, 5
7. $\models q \supset (p > q)$ — 6, MP
8. $\neg p \vee q \models p > q$ — 4, 7, classical logic
9. $p \supset q \dashv\models p > q$ — 1, 8

Responses:

- reject LI.
  - difficult.
  - you might dislike the application of LI to conditionals with logically impossible antecedents: some think that not all such conditionals are true.
  - or you might reject the explosion rule, which says that $\models (\neg p \wedge p) \supset q$ which together with LI entails $\models (\neg p \wedge p) > q$.
  - But a more complicated proof can be given which doesn't depend on those instances of LI, still yielding collapse in all but trivial cases.

`https://mandelkern.hosting.nyu.edu/IFPTHENP.pdf`

- reject IE.
  - probably the most popular option, though not often argued for explicitly.

- for instance, IE is invalid in e.g. Stalnaker/Lewis, where we get to 'forget' conditional antecedents as we go. By contrast, IE says we 'remember' successive conditional antecedents:

(1) a. If the coin is flipped, then if it lands heads, we'll win the bet.
    b. If the coin is flipped and lands heads, we'll win the bet.

(2) a. If the coin had been flipped, then if it had landed heads, we would have won.
    b. If the coin had been flipped and it had landed heads, we would have won.

(3) a. If we had ham, we would have ham and eggs if we had eggs.
    b. If we had ham and eggs, then we would have ham and eggs.

thanks to Melissa Fusco who found this example on the internet.

These look pairwise equivalent, as predicted by IE.

- what about intuitive counterexamples? they exist for subjunctives:

Etlin 2008, Yablo p.c.

(4) a. If the match had lit, then if it had been wet, it would have lit.
    b. If the match had lit and it had been wet, then it would have lit.

(5) a. If I had been six feet tall, then if I had been a bit taller than 6', I would have been 6'1".
    b. If I had been six feet tall and a bit taller than 6', I would have been 6'1".

But these examples don't seem to work for indicatives.

(6) a. If the match lit, then if it was wet, it lit.
    b. If the match lit and it was wet, then it lit.

(7) a. If I am six feet tall, then if I am a bit taller than 6', I am 6'1".
    b. If I am six feet tall and a bit taller than 6', I am 6'1".

so rejecting IE seems intuitively natural for subjunctives, but not indicatives. Even for subjunctives, there is a residual question: IE still seems to have the status of something like a natural default.

◦ so, esp. wrt indicatives, it looks attractive to consider rejecting MP. In fact, the result above relied on a peculiar application of MP, to complex conditionals. We could try to avoid the result by validating MP for simple conditionals but not complex.

- indeed, McGee argues that we have reason to do just that. Consider:

> Opinion polls taken just before the 1980 election show the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the

> other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:
>
> (8) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
>
> (9) A Republican will win the election.
>
> Yet they did not have reason to believe
>
> (10) If it's not Reagan who wins, it will be Anderson.

Or consider a die. You should plausibly have middling credence that it will land even, low credence (around $\frac{1}{3}$) that it will land 2 if prime, but maximal credence that, if it lands even, then if it lands prime, it will land 2. But, if MP is valid, your credence in the consequent should be at least your credence in the antecedent, given you're sure of the conditional.

This example is from Paolo Santorio I think.

These examples speak in favor of IE, and against MP for complex conditionals. We find similar cases in the subjunctive:

> First, suppose that a doctor and a nurse are observing a patient Jones. Jones is displaying symptoms characteristic of people who have bronchitis and are in genotype A, B, or C (say Jones has a cough and fatigue, but no fever). The doctor is trying to convince the nurse that Jones has bronchitis. In doing so, she asserts (11):
>
> (11) If Jones had had bronchitis, then if he had been in genotype A, he would be showing the symptoms he in fact is showing.
>
> But now suppose further that genotype A is negatively correlated with bronchitis: although people in genotype A can get bronchitis—in which case they display Jones's actual symptoms—most people in genotype A are immune to bronchitis. So, given his symptoms, it's most plausible that Jones has bronchitis and is in genotype B or C; and that, if he *had* been in genotype A, he would not have gotten bronchitis in the first place, and so (12) is likely false:
>
> (12) If Jones had been in genotype A, he would be showing the symptoms he in fact is showing.

An interesting difference: in the indicative cases, while the inferences in question intuitively fail to preserve truth, they still preserve certainty. Not so in the case of subjunctives.

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McGee's conclusion: we should validate IE and invalidate MP.

Furthermore, McGee shows exactly how to give a variation on Stalnaker's semantics which validates IE, while validating MP only for simple conditionals. Sentences are evaluated relative to two parameters: a Stalnakerian selection function $f$ from consistent propositions and worlds to worlds; and a 'hypothesis set' of sentences $\Gamma$, which keeps track of conditional antecedents:

Ignoring the accessibility relation.

- $[\![p]\!]^{\Gamma,w} = 1$ if $\bigcap_{r \in \Gamma} [\![r]\!]^{\varnothing} = \varnothing$; else
- $[\![A]\!]^{\Gamma,w} = 1$ iff $f(\bigcap_{p \in \Gamma} [\![p]\!]^{\varnothing}, w) \in \Im(A)$

for $A$ an atom and $\Im$ an atomic valuation

- $[\![\neg p]\!]^{\Gamma,w} = 1$ iff $[\![p]\!]^{\Gamma,w} = 0$
- $[\![p \wedge q]\!]^{\Gamma,w} = 1$ iff $[\![p]\!]^{\Gamma,w} = 1$ and $[\![q]\!]^{\Gamma,w} = 1$
- $[\![p > q]\!]^{\Gamma,w} = [\![q]\!]^{\Gamma \cup \{p\},w}$

Other systems with a similar logical profile to McGee's are given by Kratzer (1981, 1986); von Fintel (1994); Gillies (2009).

So McGee shows us how to evade the Dale/Gibbard result by validating IE, invalidating just enough of MP to block the result, while still validating all the instances of MP which were traditionally used to motivate it.

## 3 Identity

But McGee invalidates not only MP but also LI.

E.g., sentences with the form $(B \wedge \neg(A > B)) > (B \wedge \neg(A > B))$ can be false in his system, contra LI.

A sentence with this form will be true just in case the antecedent is impossible.

Intuitively: we 'remember' the antecedent in the hypothesis set; but $\neg(A > B)$ can't be true relative to a hypothesis set that entails $B$.

We could look for arguments against instances of Identity with this form. Perhaps Identity fails for complex conditionals, just as MP does.

But it doesn't, as far as I can tell:

(13) a. If Reagan will win and it's not the case that Reagan will win if Carter does, then Reagan will win and it's not the case that Reagan will win if Carter does.

b. If Reagan had won and it's not the case that Reagan would have won if Carter had, then Reagan would have won and it's not the case that Reagan would have won if Carter had.

Alternately, we could look for theories which, like McGee's, validate IE, but also validate LI. But such theories are very hard to find. Suppose we also have:

- Ad Falsum ($AF$): $\{p > q, p > \neg q\} \models \neg p$

Then we have collapse again. That is, the only connective that validates IE, LI, and AF is the material conditional:

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1. $\models (q \wedge \neg(p > q)) > \neg(p > q)$ LI
2. $\models ((q \wedge \neg(p > q)) \wedge p) > q)$ LI
3. $\models (q \wedge \neg(p > q)) > (p > q)$ IE, 2

4. $\models \neg(q \wedge \neg(p > q))$ AF, 3

5. $\neg q \models p > \neg q$ 4

6. $p, p > q, \neg q \models \neg p$ AF, 5

7. $p, p > q \models q$ reductio, 6

But 7 is MP, and now we can reproduce the Dale's result to get collapse.

Plausibly LI is beyond reproach. It's hard to see a case against AF. So then IE really has to go, whether or not we also keep MP. That is, rejecting MP alone doesn't really get us out of the result above; whether or not we reject it, we need to reject IE, too.

## 4 Bounds again

There is a parallel here to the dialectic above. To account for local compatibility and or-to-if, it looked like we wanted local information to somehow shift the interpretation of embedded conditionals. But that is hard to do without some striking peculiarities. Instead, I suggested that local info doesn't *shift* but instead *bounds* the interpretation of embedded conditionals.

That is, on the view above, even though IE is invalid and MP valid, bounds will promote certain context shifts that lead to the opposite impression.

### 4.1 Import-Export

Consider two sentences $p > (q > r)$ and $p > ((p \wedge q) > r)$. Suppose these are both satt in $\kappa$. Then they have the same truth-value at any world in $\kappa$. That is, bounds here steer us towards hearing these as equivalent. In other words, the following principle is *bounded valid*, that is, the two sides have the same truth-value at any world in any context where they are both satt:

- Lifting: $p > ((p \wedge q) > r) \dashv\models p > (q > r)$

How does that help with IE? Well, IE can be decomposed into Lifting plus another principle, Flattening:

The points about Flattening are due to Cian Dorr. Assuming logical equivalents can be substituted in antecedents of conditionals, IE is equivalent to Flattening + Lifting.

- Flattening: $p > ((p \wedge q) > r) \dashv\models (p \wedge q) > r$

These plausibly have different statuses. The subjunctive counterexamples to IE are cases where Lifting, but not Flattening, fails:

(14) a. If the match had lit and it had been wet, then it would have lit.
b. If the match had lit, then if it had lit and if it had been wet, then it would have lit.

Flattening says these are equivalent; that feels right. But (14-b) doesn't feel equivalent to (15), contrary to Lifting:

(15) If the match had lit, then it would have lit if it had been wet.

So the case we've made so far is really a case against Lifting. So Flattening may in fact be semantically valid.

And in fact we can validate it non-trivially, by adding the following constraint to Stalnaker's semantics:

- For all $\varphi, \psi : \forall w : f(\varphi \cap \psi, w) = f(\varphi \cap \psi, f(\varphi, w))$

Importantly, we must check that this constraint doesn't trivialize Stalnaker's semantics (as adding the corresponding principle which would enforce IE would).

But it doesn't. To see this constructively, take any well-order $\vec{w}$ on $\mathcal{W}$. Where $w$ is the first world in that well-order, define $f(\varphi, w)$ as the first world in $\vec{w}$ which is in $\varphi$, if $\varphi \neq \emptyset$, otherwise undefined. Now for each $u \neq w$, define $f(\varphi, u)$ as the first world in $\vec{w}$ after $u$ which is in $\varphi$, if there is one, otherwise undefined.

So Flattening will be valid for all conditionals, as desired, and Lifting bounded valid only for indicatives.

But what about the fact that subjunctives appear to obey Lifting as something like a default? Possibly subjunctives don't just lack a locality bound. Instead, all conditionals have a locality bound, but: the subjunctive mood indicates agreement with a subjunctive operator $*$, which is truth-conditionally transparent, but which expands local contexts:

From Schultheis 2020, building on a long 'modal distancing' tradition.

- $*p$ is true at $w$ iff $p$ is
- $*p$ is satt at $\kappa$ iff $p$ is satt at $\star(\kappa)$, where $\star$ is a function from sets of worlds to sets of worlds, determined by the context.

Now note that $*(p > ((p \wedge q) > r))$ and $*((p \wedge q) > r)$ are bounded equivalent; while $*(p > *((p \wedge q) > r))$ and $*((p \wedge q) > r)$ need not be. That is, to break bounded equivalence, we need an extra subjunctive operator. If the default is minimalist, we predict subjunctive Lifting to be something like a natural default.

## 4.2 Modus Ponens

Bounds thus may guide us towards interpreting invalid patterns as being valid. Conversely, they may guide us towards context shifts which make *valid* inferences appear invalid, as in the case of Modus Ponens.

An old response to McGee's examples: we interpret $q > r$ differently in $p > (q > r)$ than on its own.

But it's unclear, on existing theories, why this would be. And the basic idea is insufficiently regimented to make predictions about, e.g., the fact that MP for indicatives still appears to preserve certainty, that is, if you are certain of $p$ and of $p >_i q$, you should be certain of $q$.

Bounds give us such a story. In $p > (q > r)$, $q > r$ has local context $\kappa^p$. When we consider it on its own, it has local context $\kappa$. This will lead naturally to context-shifting. E.g.:

(16) Republican > (Not Reagan > Anderson)

(17) Not Reagan > Anderson

This gives us a story about *why* context shifts here. And it's precise enough to predict that certainty still appears to preserve MP for indicatives. For, if $p$ is true and satt throughout $\kappa$, then $\kappa^p = \kappa$. So the local context for $q > r$ will be the same in $\kappa$ as in $\kappa^p$.

The basic story for subjunctives is the same. But here the story about certainty won't apply. Suppose $p$ is true throughout $\kappa$. It doesn't follow that the local context for $q > r$ in $*(q > r)$ is the same as in $*(p > *(q > r))$: in the former, it's $\star(\kappa)$: in the latter, $\star(\kappa)^p$. Since $\star(\kappa)$ doesn't necessarily entail $p$ anymore, it need not be the same as $\star(\kappa)^p$. And this is intuitive as a story about why MP appears not to preserve certainty for subjunctives.

'If John is genotype A, then if he has bronchitis, he would be showing the symptoms he in fact is.'

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