Day 4: Tenability, Contextualism, and Update

Conditionals, probability, and decision // *ESSLLI 2024*
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1 More on Tenability and Triviality

Mock-up of Lewis's Proof

See Lewis (1976, pg. 300). As it is written in his paper, the proof is more compressed than the version I give here.

1. $Pr(A > B) =$ (by Law of Total Probability (LTB))
2. $Pr(A > B \mid B)Pr(B) + Pr(A > B \mid \neg B)Pr(\neg B) =$ (by notation)
3. $Pr_B(A > B)Pr(B) + Pr_{\neg B}(A > B)Pr(\neg B) =$ (by ST)
4. $Pr_B(B \mid A)Pr(B) + Pr_{\neg B}(B \mid A)Pr(\neg B) =$ (by Ratio Formula)
5. $\left(\frac{Pr_B(AB)}{Pr_B(A)}\right)Pr(B) + \left(\frac{Pr_{\neg B}(AB)}{Pr_{\neg B}(A)}\right)Pr(\neg B) =$ (by algebra)
6. $\left(\frac{Pr_B(A)}{Pr_B(A)}\right)Pr(B) + \left(\frac{Pr_{\neg B}(AB)}{Pr_{\neg B}(A)}\right)Pr(\neg B) =$ (by algebra)
7. $1 \cdot Pr(B) + 0 \cdot Pr(\neg B) = Pr(B)$ ✓

The contextualist rejects the move from 2 to 3. A helpful bit of notation distinguishes

Like Bacon (2015).

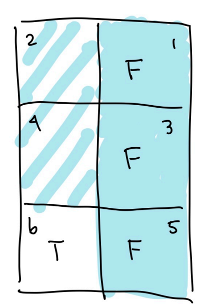
A way to put this: Stalnaker's Thesis does not hold if the space of rational credence functions is closed under conditioning on (nonconditional) propositions

$$Pr(A >_{Pr} B) \quad (1)$$

$$Pr(A >_{Pr_B} B) \quad (2)$$

For another window on why, consider an example (from Goldstein and Santorio, 2021). Intuitively, the probability of (1) is 1/4:

(1) If [this fair] die does not land (two or four), then it will land six.



This is the (ST)-compliant conditional probability $Pr(6 \mid \neg(2 \vee 4))$. In addition—again, intuitively—the probability of (2) and (3) are each 1/2:

(2) The die will land even.

(3) The die will land odd.

Suppose now that we know we'll learn whether the die landed even or odd. I say to you:

(4) If the die lands even, then (if it does not land (two or four), it will land six).

What do you suppose is the probability of (4)? A natural answer is 1: (4) is certainly true.

In addition,

(5) If the die lands odd, then (if it does not land (two or four), it will land six).

appears to be certainly *false*: it has an apparent probability of 0.

Recall that the Law of Total Probability (LTP) entails that $Pr(\phi) = Pr(\phi \mid X)Pr(X) + Pr(\phi \mid \overline{X})Pr(\overline{X})$

2 Three Equations

Goldstein and Santorio (2021) also helpfully caution us to distinguish three things:

- **Stalnaker's Thesis.** $Pr(A > B) = Pr(B \mid A)$

  ⇒ a thesis about semantics.

- **The Update Thesis.** The rational posterior credence to have in (arbitrary) B, if you learn all and only A, is $Pr(B \mid A)$ (where $Pr(\cdot)$ is your prior.)

  ⇒ a thesis from confirmation theory; a thesis about epistemic rationality.

- **The Ratio Formula.** $Pr(B \mid A) = \frac{Pr(BA)}{Pr(A)}$

  ⇒ an analytic notational convention.

A new notation for the normative quantity: $Pr^{[+A]}(B) = Pr(B \mid A)$

Combining *all* of these gives us something that does not seem satisfiable:

$$Pr(A > B) = Pr(B \mid A) = Pr^{[+A]}(B)$$

in a case like the die case, where (it is important to note) B is itself a conditional (viz., (1)).

3 Kaufmann and the Ramsey Test

Here is a paraphrase of the Ramsey test (1978/1931):

> For any individual, the acceptability of a conditional $(A > B)$ is the degree to which she would accept B on the supposition that A, provided A is epistemically possible for her.

At first pass, it looks like Stalnaker's Thesis should support this. But Kaufmann (2004) makes some interesting observations in the vicinity. Consider any credally represented partition $\{X, Y\}$. Kaufmann notes that the following is true:

Equation (3) follows from a generalization of LTP:

$$Pr(X) = \sum_J Pr(J) \sum_K Pr(K \mid J) Pr(X \mid KJ)$$

Note also that (3) can also be written $Pr(B \mid A) = Pr(X \mid A)\, Pr_X(B \mid A) + Pr(Y \mid A)\, Pr_Y(B \mid A)$

$$Pr(B \mid A) = Pr(X \mid A)\,Pr(B \mid AX) + Pr(Y \mid A)\,Pr(B \mid AY) \tag{3}$$

And also notes that the following is emphatically *not* (in general) true:

$$Pr(B \mid A) = Pr(X)\,Pr(B \mid AX) + Pr(Y)\,Pr(B \mid AY) \tag{4}$$

Here's a simplified example that gets at the difference between Equations (3) and (4).

> **Knights and Knaves**. You have a strange rash which can only be caused by mutually exclusive diseases X and Y. You consult two doctors. Unfortunately, one is a knight and the other is a knave. Doctor 1 says: "Probably you have X. If you have X, you should take aspirin." Doctor 2 says: "Probably you have Y." As you leave, he adds: "But for what it's worth, if you *do* have X after all, you should *not* take aspirin!"

If your credences obey Stalnaker's Thesis, then your credence in $\ulcorner X \to S \urcorner$ is around .5 ("S" for "aspirin"). But if you (were to) learn X, your credence in S goes (would go) up considerably past .5, *in violation of the Ramsey Test*. Why? Because learning X strongly confirms that it was Doctor 1, not Doctor 2, who was the knight. Equation (3) describes this. Kaufmann describes the psychological step of updating your priors on $\{X, Y\}$ as "abduction". So one way to frame the question is whether we are rationally required to perform abduction when we assign credences to conditionals.

A way this is sometimes put in the opinion aggregation literature is that *pooling* does not commute with conditionalization: if you have a committee of putative experts and you are taking a weighted linear average of their opinions on a matter, new evidence will affect the weighting itself.

An important connection: our definition of imaging from Day 1 corresponds to Kaufmann's *local* reading, viz., to *not* performing the abductive step. CDT taps a local reading of a (weighted sum of) conditionals ($A > \$n$), EDT taps a global reading of a (weighted sum of) conditionals ($A > \$n$). In a Newcomb-like case:

Kaufmann says no! He claims there are "local" readings of the indicative conditional corresponding to (4), and "global" (ST-compliant) readings corresponding to (3).

At any w, the proposition $\$n$ is true iff the agent's utility in w is n.

	K1	K2
1B	\$1M	\$0M
2B	\$1.1M	\$0.1M

$$\begin{aligned} CEU(A) &= \sum_{n \in \mathbb{N}} Pr(A >_l \$n) \\ &= \sum_{n \in \mathbb{N}} [\,Pr(K_1)\,Pr(\$n \mid K_1 A) + Pr(K_2)\,Pr(\$n \mid K_2 A)] \end{aligned}$$

$$\begin{aligned} EEU(A) &= \sum_{n \in \mathbb{N}} Pr(A >_l \$n) \\ &= \sum_{n \in \mathbb{N}} [\,Pr(K_1 \mid A)\,Pr(\$n \mid K_1 A) + Pr(K_2 \mid A)\,Pr(\$n \mid K_2 A)] \end{aligned}$$

4 Chance: Theory, Practice, Models

A natural application of Kaufmann's observation takes $\{X, Y\}$ to be a causal or chance partition: viz., $X = [Ch = \pi]$ for some particular chance-candidate π (a probability function).

Some ideas about chance (from Lewis, 1980 and subsequent literature):

- **screening-off**. If you are rational, knowledge of chances screens off all other information. For example: if you know that the objective bias of this coin is .75 in favor of heads, you don't need to know anything else—where it came from, what it's made of, how many times it's been flipped—to know that it's rational to believe to degree .75 that it will come up heads when flipped.

Lewis worries that there were some exceptions to this, involving oracles and time-travelers. He called this "inadmissible information."

- **laws of nature**. Laws of nature may invoke objective chances. For example, ^{17}N (a nitrogen isotope) has a half-life of about 4 seconds. This is often glossed in the chance literature as a statement about chance: $Ch(x \text{ decays in the next 4 seconds} \mid x \text{ is an atom of } ^{17}N) = 1/2$.
- **Lewis's Principal Principle.** Here are three versions in ascending order of complexity:
  1. $Pr_t(R \mid E \wedge [Ch_t(R) = x]) = x$
     - Spencer (2020) calls this "the Present Principle".
  2. $Pr_0(R \mid [Ch_t(R) = x]) = x$
  3. If Q is admissible with respect to $[Ch_t(R) = x]$, then $Pr_0(R \mid Q \wedge [Ch_t(R) = x]) = x$.

The bridge from #1-#2 can be supported by the thought that $Pr_t(\cdot) = Pr_0(\cdot \mid E)$, where E is the total information learned since the agent's "epistemic birth".
The bridge from #2-#3 Assumes inadmissible information is impossible.

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