Witnesses

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1 Two faces of indefinites

In the classical picture, indefinites are existential quantifiers, definites are existential quantifiers with uniqueness, and pronouns are variables.

This accounts well for one aspect of indefinites: their existential import. E.g., the interaction of negation with indefinites is strong:

(1) Sue doesn't have a kid. \equiv Sue is childless.

And referential intention doesn't seem to matter to truth.

But it fails to account for how indefinites license subsequent definites:

- (2) a. Sue has a kid. She lives at a boarding school.b. Sue is a parent. She lives at a boarding school.
- (3) a. Robin has a twin. He lives in Dubuque.b. Robin is a twin. He lives in Dubuque.
- (4) a. Latif rode a bike to work. It was heavy.b. Latif biked to work. It was heavy.

A helpful generalization: indefinites have open scope to their right.

$$3x(p \wedge q) \equiv 3xp. \ \iota x(p,q). \equiv 3xp \wedge \iota x(p,q)$$

Hence (2-a)'s prominent reading is 'Sue has a kid at boarding school', while (2-b)'s is 'Sue is a parent at boarding school'.

We could say that, e.g., the indefinite in (2-a) takes scope over the pronoun in the second sentence. But this doesn't help with quantified versions:

- a. Everyone who has a kid loves { them the kid }
 b. Every parent loves { them the kid }.
- a. Everyone who rode a bike to work sold { it the bike }.
 b. Everyone who biked to work sold { it the bike }.
- a. Everyone who has a cat loves { her the cat }.
 b. Every cat-owner loves { her the cat }.

Consider (7-a). Flatfootedly, that should get the truth-conditions:

(8) $\forall x((\exists y(Cy \land H(x,y))) \rightarrow L(x,y))$

But the y on the RHS is unbound. If we give the indefinite wide scope we get absurdly weak truth-conditions:

Frege/Russell/Strawson. Following presentation in Heim 1982; Cumming 2015.

Variants on Partee's marble sentence reported in Heim 1982.

Egli 1979. I use 3 for the indefinite and t for the definite, reserving \exists for the existential quantifier. I treat pronouns as definites with tautological restrictors.

Note there are two issues here: accounting for the extant reading, and for the contrast when the indefinite is missing.

2 Existing approaches

One can think of each of the two main threads in the literature as focusing on one of these aspects of indefinites and trying to explain away the other.

E-type theories try to defend the classical picture of (in)definites, adopting non-classical theories of everything else.

Dynamic theories instead treat indefinites as variable licensers and locate their existential force in other parts of the grammar.

Both approaches have well-known problems. E-type theories require nonstandard treatments of connectives and pragmatics which have not been adequately worked out. They have well-known problems with "indistinguishable participants". I think they also fail to account for contrasts like these:

(10) a. Everyone who has a twin loves the twin.

b. Every twin loves the twin.

On dynamic theories, indefinites update *variable assignments*. Contexts are treated as sets of partial assignment-world pairs, accounting for the existential import of asserted indefinites. So e.g. $3x \ kid-of-Sue(x)$ takes a context *c*, checks that *x* is novel (nowhere defined) in *c*; "frees" *x* throughout *c*, and then retains pairs $\langle g, w \rangle$ s.t. g(x) is a kid of Sue's in *w*:

$$- c[3x] = \begin{cases} \# & \exists \langle g, w \rangle \in c : g(x) \neq \# \\ \{ \langle g, w \rangle : \exists g' : \langle g', w \rangle \in c \land g >_x g' \} & otherwise \end{cases}$$
$$- c[P(x_1, \dots, x_n)] = \{ \langle g, w \rangle \in c : \langle g(x_1), \dots, g(x_n) \rangle \in \mathcal{I}(P, w) \}$$

Definites require that their variable be defined throughout the context (*familiar*) and assigned to a restrictor-thing throughout the context. Where defined, they are just like the corresponding open sentence:

$$- c[\iota x(p,q)] = \begin{cases} \# & c[p] \neq c \\ c[q] & otherwise \end{cases}$$

We treat pronouns as definites with tautological restrictors, so that the requirement is simply that x be defined throughout the context.

Conjunction and successive assertion are both successive update:

$$- c[p \land q] = c[p, q] = c[p][q]$$

This captures the open scope of indefinites:

$$c[3xFx. \iota x(Fx, Gx)] = c[3xFx \land \iota x(Fx, Gx)] = c[3x(Fx \land Gx)]$$

Evans 1977; Heim 1990 etc.

and definites as corresponding variables; Kamp 1981; Heim 1982; Groenendijk and Stokhof 1991; Chierchia 1992; Dekker 1994; Beaver 2001 etc. I will present a simplified version of Heim's semantics.

Mandelkern and Rothschild 2020

 $g >_x g'$ iff g and g' agree everywhere except on x, where g is defined and g' is not.

e.g. $\iota x(\top x, q)$, where $\forall w : \mathcal{I}(\top, w) = D$.

But what about the strong meanings of negated indefinites? Now that indefinites aren't quantificational, we can't treat negation as complementation. Instead, we make negation a universal quantifier over assignments:

$$c[\neg p] = \{ \langle g, w \rangle \in c : \nexists g' \ge g : \langle g', w \rangle \in c[p] \}$$

This gets the quantificational force of negated indefinites right, but it leads to problems: double negation kills anaphoric potential, since universal quantifiers aren't involutive.

E.g., 'It's not the case that Sue doesn't have a kid' will take *c* to the set of points $\langle g, w \rangle \in c$ s.t. Sue is childless in *w*, putting no constraints on the *g*'s.

This is not *necessarily* a problem; after all, the facts about anaphora, in particular the open scope of indefinites, suggest that some part of classicality needs to be discarded. But it is *in fact* a problem, because as Karttunen (1976) (sort of) observed, doubly negated indefinites license definites:

- (11) a. Sue doesn't have a kid!
 - That's not true! She's at boarding school.
 - b. Sue isn't a parent!- That's not true! ??She's at boarding school.

More worryingly, this problem infects disjunction. Compare:

(12) a. Either Sue doesn't have a kid, or she lives at boarding school.b. Either Sue isn't a parent, or she lives at boarding school.

The natural way to try to account for bathroom sentences is to say that the negation of the left disjunct is available to license material in the right disjunct:

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$$c[p \lor q] = c[p] \cup c[\neg p][q]$$

Then

$$c([\neg 3xFx) \lor \iota(Gx, Fx)] = c[\neg 3xFx] \cup c[\neg \neg 3xFx][\iota x(Gx, Fx)]$$

But since $\neg \neg$ doesn't cancel out, this doesn't give what we want, namely $c[3xFx][\iota x(Gx,Fx)]$.

So, for instance, (12-a) is predicted to be defined in c only if there is an antecedently available referent for 'she'.

3 A simple trivalent approach

The challenge: *capturing both aspects of indefinites*. Ignoring their anaphoric potential and trying to mess with the rest won't work. But nor will hiving off existential import into negation. I want instead to explore the idea that both aspects of indefinites are *different aspects of a single meaning of indefinites*.

Then $c[\neg_3 xp]$ would invariably be \emptyset or undefined.

 $g' \ge g$ iff g' has the same value as g everywhere that g is defined.

A quick way to see this: for any $p, c[\neg p]$ is a *subset* of c, while c[3xp], wherever defined, is always a pointwise extension of a subset of c.

Evans 1977, and Partee, reported by Roberts 1989; following Partee's example these are often called *bathroom disjunctions*.

Beaver 2001

Cf. Krahmer and Muskens 1995's bilateral dynamic approach, as well as van den Berg 1996; Schlenker 2011; Chatain 2017; Gotham 2019; Hofmann 2019; Elliott 2020. My own account is especially indebted to discussion with Keny Chatain and his insightful notes. A simple idea for such a theory would start with a trivalent entry like this:

$$[[3xp]]^{g,w} = \begin{cases} 1 & [[p]]^{g,w} = 1 \\ 0 & \forall g'[x]g : [[p]]^{g',w} = 0 \\ \# & otherwise \end{cases}$$

This captures the spirit of the idea I want to develop. But as Spector (2021, citing Anvari) insightfully points out, it doesn't work, because sometimes indefinites in positive environments need to allow assignment variation, too. This isn't obvious for assertions but it shows up in cases like this:

(13) a. There isn't a cat that doesn't have a hat. b. $\neg 3x(Cx \land \neg 3yH(x,y))$

This trivalent view predicts that whether an indefinite has quantificational or referential meaning depends on the polarity of its environment, so the doubly negated indefinite *3y* has a referential reading and (13-a) can only mean: there is a hat which every cat has.

4 A two-dimensional approach

My account keeps the classical *truth conditions*, and captures anaphoric relations in a separate dimension.

Both dimensions are part of the meaning of indefinites, but only the first is targeted by embedding operators.

So 3xp has the *truth/falsity*-conditions of $\exists xp$. But it also requires, in a separate dimension of meaning, that it is *witnessed* if true. That is, $[\exists xp]^{g,w}$ is

- satt only if $[\exists x p]^{g,w} = 1 \rightarrow [p]^{g,w} = 1$
- true iff $[\exists x p]^{g,w} = 1$, false otherwise

So, e.g. $3x \ kid \ of \ Sue(x)$ is

- true at $\langle g, w \rangle$ iff Sue has a kid in w, false otherwise;
- satt at $\langle g, w \rangle$ iff, IF Sue has a kid in w, then g(x) is Sue's kid in w.
 - \hookrightarrow So, it is true and satt iff g(x) is Sue's kid in w.
 - \hookrightarrow It is false and satt iff Sue is childless in w.

This matches the extension/anti-extension we were going for with the simple trivalent theory. But we can already see how it can avoid the Spector/Anvari problem, because *connectives get their classical truth/falsity conditions*, and the *truth-conditional contribution* of the indefinite, which is what is targeted by negation, is always quantificational.

Contexts are sets of pairs of (possibly partial) variable assignments and worlds, just as in dynamic semantics; given a context κ , updating with p results in $\kappa^p = \{\langle g, w \rangle \in \kappa : p \text{ is true and satt at } \langle \kappa, g, w \rangle\}.$

While I will explore *complicating semantic* values in response to this, Spector (2021) explores instead *complicating assignment func*-

tions.

I call this second dimension *bounds*. I write 'satt' for 'has its bounds satisfied'.

the witness bound. \exists is the classical existential quantifier. 'Only if' because this leaves out projection conditions for *p*'s bounds, which we can mostly ignore.

This eliminative approach does not require bounds to be satisfied throughout the input context, pace Stalnaker etc. This is important: it is only this that allows indefinites to license new variables.

g'[x]g iff g' agrees with g everywhere except possibly on x, where g' is defined. See Mandelkern ta.

Definites require familiarity; $[x(p,q)]^{\kappa,g,w}$ is

- true iff $[p \land q]^{g,w} = 1$
- satt only if $\forall \langle g', w' \rangle \in \kappa : [\![p]\!]^{\kappa, g', w'}$ is true and satt

Finally, we need to say how bounds project through connectives:

- $p \wedge q$ is satt at $\langle \kappa, g, w \rangle$ iff p is satt at $\langle \kappa, g, w \rangle$ and q is satt at $\langle \kappa^p, g, w \rangle$
- $p \lor q$ is satt at $\langle \kappa, g, w \rangle$ iff p is satt at $\langle \kappa, g, w \rangle$ and q is satt at $\langle \kappa^{\neg p}, g, w \rangle$
- $\neg p$ is satt at $\langle \kappa, g, w \rangle$ iff *p* is

4.1 An example and some observations

To get the intuition, suppose we update a context κ with (14):

(14)
$$3x \, kid \cdot of \cdot Sue(x)$$

We keep all points $\langle g, w \rangle \in \kappa$ where (14) is true and satt:

- *truth*: Sue must have a kid in w; and
- *satt*: *if* Sue has a kid in w, g(x) is Sue's kid in w.

So the *update effect* of (14) is to keep exactly the points $\langle g, w \rangle \in \kappa : g(x)$ is Sue's kid in w.

Suppose then we update the resulting context κ' with (15):

- (15) $\iota x(kid(x), at-boarding-school(x))$
- This is satt throughout κ' , since for every point $\langle g, w \rangle \in \kappa'$, kid(x) is true at $\langle g, w \rangle$ (since we've updated with the corresponding indefinite).
- It is true at a point $\langle g, w \rangle \in \kappa'$ iff g(x) is at boarding school in w.

Hence indefinites pave the way for subsequent definites.

In general, the sentences in (16) are equivalent in the sense that *each is satt* and true iff all the others are (iff $g(x) \in \mathcal{I}(F, w) \cap \mathcal{I}(G, w)$):

(16) a. $3xFx \wedge \iota x(Fx, Gx)$ b. $3x(Fx \wedge Gx)$ c. $3xFx. \iota x(Fx, Gx).$

But without a preceding indefinite, definites won't be anywhere satt; e.g. 'Sue is a parent. The kid is at boarding school' will lead to a crash.

Negated indefinites have a strong meaning, since again negation only targets the *truth-conditions* (not bounds) of the indefinite. So $\neg 3x \, kid\text{-of-Sue}(x)$

- is *true* at $\langle g, w \rangle$ iff Sue is childless in w

Hence we are now relativizing semantic values to a context parameter too. I actually prefer a uniqueness approach which subsumes this one, but this version makes for a more minimal comparison with other systems. Pronouns are again definites with tautological restrictors, hence just require that their variable is defined throughout the context.

There are both symmetric and asymmetric versions of these. We could also eliminate the recursive definitions and replace them with pragmatic rules, building on Schlenker 2008, as Spector (2021) suggests.

'Sue has a child'.

So updating with an indefinite sentence is just like updating with the corresponding open sentence.

'The kid is at boarding school.'

3x(parent(x)). at-boarding-school($\iota x(kid x)$).

- and satt at $\langle g, w \rangle$ iff either Sue is childless in w or g(x) is Sue's child in w.

We can confirm that $\neg_{3x}(Cx \land \neg_{3y}H(x,y))$ gets the desired truth-conditions.

Double negation elimination is valid, since the truth conditions of negation are classical and bounds project through negation. Hence bathroom disjunctions work smoothly:

(17)
$$\neg 3x \ kid-of-Sue(x) \lor \iota x(kid(x), at-boarding-school(x)).$$

Consider a point $\langle g, w \rangle$:

- If Sue is childless at *w*, then the sentence is true and satt, since the witness bound is trivially satisfied; and the familiarity bound of the right disjunct is satisfied in its local context, which will only contain points where *3x kid-of-Sue(x)* is true and satt.
- If Sue has a child at w, the sentence is only satt if g(x) is her child, thanks to the witness bound, which projects from the left disjunct; and true iff g(x) is at boarding school.

4.2 Quantifiers

Quantifiers are indexed to an individual variable as well as a domain variable: $every_{x,\alpha}(p,q)$ is

- satt at $\langle \kappa, g, w \rangle$ iff $g(\alpha)$ is a non-empty set comprising assignments g' which
 - agree with g except possibly on x and variables novel in κ , and
 - are s.t. $p \land q$ is satt at either $\langle \kappa, g', w \rangle$ or $\langle \kappa_{\alpha}, g', w \rangle$, where $\kappa_{\alpha} = \{ \langle g', w' \rangle : \exists g'' : \langle g'', w' \rangle \in \kappa \land g' \in g''(\alpha) \}$
- true iff everything assigned to *x* by an element of $g(\alpha)$ which makes *p* true is assigned to *x* by an element of $g(\alpha)$ which makes $p \wedge q$ true

5 Comparison to Heim Ch. 2, Rothschild

A different trivalent approach offloads existential closure, not to negation, but rather to a covert operator †:

$$- [[3xp]]^{g,w} = \begin{cases} 1 & [[p]]^{g,w} = 1 \\ 0 & g(x) = \# \lor [[p]]^{g,w} = 0 \\ \# & otherwise \end{cases}$$

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$$[P(x_1...x_n)]^{g,w} = \begin{cases} \# & \exists i \in [1,n] : g(x_i) = \# \\ 1 & \langle g(x_1), \dots g(x_n) \rangle \in \mathcal{I}(P,w) \\ 0 & \langle g(x_1), \dots g(x_n) \rangle \notin \mathcal{I}(P,w) \end{cases}$$

Heim 1982, Ch. 2; Rothschild 2017, and Heim's Stockholm handout. The view I present here is Rothschild's, whose paper was enormously helpful for me in understanding the theoretical possibilities here.

novel now means: receiving any possible

This disjunctive condition allows for quantificational subordination. Other quantifiers can

value, including undefined

be built on this schema.

and is trivially satt.

 $- [[\dagger p]]^{g,w} = \begin{cases} \# & [[p]]^{g,w} = \# \\ 1 & \exists g'[AS(p)]g : [[p]]^{g',w} = 1 \\ 0 & otherwise \end{cases}$

¬ is Boolean, ∧ middle Kleene semantics. Pronouns are free variables, with $[x]^{g,w} = g(x)$ where defined and otherwise undefined.

Finally, Rothschild assume Stalnaker's bridge: a sentence is assertible only if it is defined throughout the context.

We can parse the Anvari/Spector sentence as $\neg \dagger 3x(Cx \land \neg \dagger 3yH(x,y))$, giving indefinites quantificational force *whether in positive or negative environments* (as needed).

My worry is that, while this system has enough expressive power to capture the readings we want, it overgenerates readings:

(18) a. There isn't a cat_x .

b. $\neg 3xCx$

c. $\neg \dagger 3xCx$

(18-a) has a prominent reading as (18-c). But it doesn't have a reading as (18-b), which says: g(x) is either undefined or not a cat.

In her recent handout, Heim suggests we can rule out such readings with a notion of *assertoric content*: $\{w : [[+p]]^{\emptyset,w} = 1\}$. And the assertoric content of (18-b) is trivial, sufficing to rule out this reading.

I think this response is not available in some more complex cases. Consider:

(19) a. There is a cat and there isn't a cat. b. $\dagger(3xCx \land \neg 3yCy)$.

This is a non-trivial assertoric content: it says that something is a cat.

Maybe the theory can be localized: the second conjunct here doesn't add anything. More complex cases, however, won't permit a similar response:

(20) a. Either there isn't a cat or it is Bengal, and I don't own something. b. $+((\neg +3xCx \lor (3xCx \land Bx)) \land \neg 3xOx).$

The assertoric content is: either there are no cats, or there is a Bengal not owned by me. The final conjunct contributes non-trivially to this content. But this is plainly not a reading of (20-a).

So I don't think pragmatic considerations will suffice to rule out unwanted readings.

This example also brings out a worry about deriving the Novelty constraint (indefinites can't be co-indexed with indefinites to their left) from Maximize Presupposition. I think disjunction will again pose a problem for this idea. Here's a simpler case to bring this out:

(21) Either there's a_x cat upstairs or Mark will be sad.

g'[AS(p)]g iff g' agrees with g except possibly on the "assignment sensitive" variables in p, essentially those bound by an indefinite in p; Rothschild gives a clever semantic characterization of this.

The extra complexity of the disjunction means that Maximize Presupposition won't rule out this parse, a point I return to presently. Maybe if we stipulate Novelty syntactically, we'll be able to pragmatically rule out unavailable readings. After updating with (21), *x* will be defined in *some but not all* of the remaining assignments, so the definite would not be licensed here (nor would the open sentence). Nonetheless, co-indexation of indefinites should not be allowed here anymore than in other cases.

(22) Either there's a cat upstairs or Mark will be sad. Susie likes a cat.

This doesn't have a reading where it means that either there's a cat upstairs that Susie likes, or else Mark will be sad and Susie likes some cat or other.

You could add Novelty as a semantic presupposition to the system, but you'd need to localize it to local contexts, and then the architecture looks less minimal.

Novelty is easy to add a presupposition to the bounded system, where we already have local contexts—or just as an entailment. While I share the intuition that novelty and familiarity are somehow complementary, hence interderivable, cases of disjunction show that this is hard to make good on.

A final worry: how are definites in right disjuncts licensed? Rothschild suggests (23-a) gets the parse in (23-b):

- (23) a. Either there's not a_x bathroom or it_x's upstairs.
 - b. $\neg \dagger 3xBx \lor (3xBx \land \iota xUx)$

This gets the right truth-conditions via the addition of extra syntactic material. Rothschild's constraint: the added material can't change the classical truthconditions of the sentence—looks a lot like Schlenker (2008)'s local contexts.

If you're worried about the syntax, then you'll want to switch to semantically or pragmatically calculated local contexts. The negation of the left disjunct thus needs to make x familiar. But then the proposed decomposition doesn't work; we need instead an extension/anti-extension roughly like mine.

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These kinds of cases also pose a problem for what Heim calls *Uniformity*, which says that all the assignments in a context have the same domain.

You can't have Novelty as a syntactic stipulation consistent with Rothschild's treatment of bathroom disjunctions.

We could say that indefinites aren't really indexed, but rather always predicate their complement of the denotation of the context's minimal novel variable.

Lots of other interesting recent systems I haven't talked about, from Patrick Elliott, Lisa Hofmann, Keny Chatain, Benjamin Spector, etc.

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